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Heavy-Meson Observables at One-Loop in Partially Quenched Chiral Perturbation Theory

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Abstract

I present one-loop level calculations of the Isgur-Wise functions for $B \to D^{(*)} e\nu$, of the matrix elements of isovector twist-2 operators in B and D mesons, and the matrix elements for the radiative decays $D^* \to D\gamma$ in partially quenched heavy quark chiral perturbation theory. Such expressions are required in order to extrapolate from the light quark masses used in lattice simulations of the foreseeable future to those of nature.

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I. INTRODUCTION

In order to extract fundamental quantities from systems containing heavy quarks, such as the weak mixing angles V_{bc} and V_{bu}, the strong interaction contributions to the observable of interest must be determined. This is because the standard model of electroweak interactions is constructed in terms of quarks, gluons and leptons, while we are only able to study the quarks via the properties of the hadrons within which they are confined. Significant progress was made in eliminating the strong interaction uncertainties in some matrix elements in hadrons containing one or more heavy quarks in the late eighties with the discovery of the heavy quark symmetries [1], and the formulation of heavy quark effective theory [2]. Ultimately, such strong interaction effects will be directly computable from numerical lattice QCD simulations, however, at this point in time, and in the foreseeable future, lattice simulations will not be performed with the physical values of the light-quark masses, m_q. In order to make a connection between the lattice simulations and nature, an extrapolation from the lattice quark masses to those of nature is required. One can most efficiently perform the extrapolation for many observables using simulations of partially quenched QCD (PQQCD) matched to partially quenched chiral perturbation theory (PQχPT) [3–9]. Some observables have been analyzed with PQχPT in the light pseudo-scalar meson sector, such as decay constants and masses [5]. Further, the decay constants and B-parameters for mesons containing a single heavy quark have been computed [4]. In this work we present the one-loop analysis in PQχPT of the Isgur-Wise function for $B \to D^{(*)}$ decays, of the matrix elements of isovector twist-2 operators that are directly related to moments of the parton distribution functions, and of the radiative decays $D^* \to D\gamma$.

II. PQQCD

We will consider a theory constructed from three valence-quarks, u, d, s, three-sea quarks j, l, r and three bosonic-quarks $\tilde{u}, \tilde{d}, \tilde{s}$. The masses of the bosonic-quarks are equal to those of the valence-quarks, $\overline{m}, \overline{m}, m_s$ respectively, where we have chosen to work in the isospin limit. The sea-quarks are taken to be degenerate with mass m_j. The valence-, sea-, and bosonic-quarks are combined into column vectors

$$Q_L = \left(u, d, s, j, l, r, \tilde{u}, \tilde{d}, \tilde{s}\right)^T_L \quad , \quad Q_R \ = \ \left(u, d, s, j, l, r, \tilde{u}, \tilde{d}, \tilde{s}\right)^T_R \quad , \tag{1}$$

which transform in the fundamental representation of $SU(6|3)_{L,R}$ respectively. The ground floor of Q_L transforms as a $(\mathbf{6}, \mathbf{1})$ of $SU(6)_{qL} \otimes SU(3)_{\tilde{q}L}$ while the first floor transforms as $(\mathbf{1}, \mathbf{3})$. The right handed field Q_R transforms analogously. The PQQCD Lagrange density is invariant under $SU(6|3)_L \otimes SU(6|3)_R$ chiral transformations, and in direct analogy with QCD a chiral Lagrangian can be constructed to describe the low-energy dynamics of the low lying hadrons.

The low-energy QCD dynamics of hadrons containing a single heavy quark are described by a chiral lagrangian that has the heavy quark spin symmetry and flavor symmetry manifest [10,11]. The extension of the heavy quark chiral Lagrange density to describe heavy quark systems in PQQCD can be found in Ref. [4]. The dynamics of the pseudo-scalar mesons are described at leading order by a Lagrange density of the form,

$$\mathcal{L} = \frac{f^2}{8}\mathrm{str}\left[\ \partial^\mu \Sigma^\dagger \partial_\mu \Sigma\ \right] \ +\ \lambda\ \mathrm{str}\left[\ m_Q \Sigma^\dagger + m_Q \Sigma\ \right] \ +\ \alpha_\Phi \partial^\mu \Phi_0 \partial_\mu \Phi_0 \ -\ m_0^2 \Phi_0^2 \quad , \qquad (2)$$

where the meson field is incorporated in Σ via

$$\Sigma = \exp\left(\frac{2\ i\ \Phi}{f}\right) \ =\ \xi^2 \quad , \quad \Phi \ =\ \begin{pmatrix} M & \chi^\dagger \\ \chi & \tilde{M} \end{pmatrix} \quad , \qquad (3)$$

where M and $\tilde{M}$ are matrices containing bosonic mesons while χ and $\chi^\dagger$ are matrices containing fermionic mesons, with

$$M = \begin{pmatrix} \eta_u & \pi^+ & K^+ & J^0 & L^+ & R^+ \\ \pi^- & \eta_d & K^0 & J^- & L^0 & R^0 \\ K^- & \overline{K}^0 & \eta_s & J_s^- & L_s^0 & R_s^0 \\ \overline{J}^0 & J^+ & J_s^+ & \eta_j & Y_{jl}^+ & Y_{jr}^+ \\ L^- & \overline{L}^0 & \overline{L}_s^0 & Y_{jl}^- & \eta_l & Y_{lr}^0 \\ R^- & \overline{R}^0 & \overline{R}_s^0 & Y_{jr}^- & \overline{Y}_{lr}^0 & \eta_r \end{pmatrix} \quad , \quad \tilde{M} \ =\ \begin{pmatrix} \tilde{\eta}_u & \tilde{\pi}^+ & \tilde{K}^+ \\ \tilde{\pi}^- & \tilde{\eta}_d & \tilde{K}^0 \\ \tilde{K}^- & \tilde{\overline{K}}^0 & \tilde{\eta}_s \end{pmatrix}$$

$$\chi = \begin{pmatrix} \chi_{\eta_u} & \chi_{\pi^+} & \chi_{K^+} & \chi_{J^0} & \chi_{L^+} & \chi_{R^+} \\ \chi_{\pi^-} & \chi_{\eta_d} & \chi_{K^0} & \chi_{J^-} & \chi_{L^0} & \chi_{R^0} \\ \chi_{K^-} & \chi_{\overline{K}^0} & \chi_{\eta_s} & \chi_{J_s^-} & \chi_{L_s^0} & \chi_{R_s^0} \end{pmatrix} \quad . \qquad (4)$$

where the upper 3×3 block of M is the usual octet of pseudo-scalar mesons while the remaining entries correspond to mesons formed with the sea-quarks. The singlet field is defined to be $\Phi_0 = \mathrm{str}\,(\ \Phi\ )/\sqrt{2}$, and its mass m_0 can be taken to be of order the scale of chiral symmetry breaking, $m_0 \to \Lambda_\chi$ [9]. The super mass matrix, m_Q, is

$$m_Q = \mathrm{diag}\,(\overline{m}, \overline{m}, m_s, m_j, m_j, m_j, \overline{m}, \overline{m}, m_s) \quad , \qquad (5)$$

where we will work in the limit of exact isospin symmetry. The convention we use corresponds to $f \sim 132$ MeV.

The B-mesons with quantum numbers of $b\overline{Q}$ form a nine-component vector,

$$B = (B_u, B_d, B_s, B_j, B_l, B_r, B_{\tilde{u}}, B_{\tilde{d}}, B_{\tilde{s}}) \quad , \qquad (6)$$

and heavy quark spin symmetry (for a comprehensive review see Ref. [16]) is implemented by combining the annihilation operators for the B and B^* mesons together into the field H_v

$$H_v = \frac{1}{2}(1 + \not{v})\left[\ \gamma^\alpha B_\alpha^* + i\gamma_5 B\ \right] \quad , \qquad (7)$$

where $H_v \to H_v V^\dagger$ under $SU(3)$ flavor transformations and $H_v \to S_b H_v$ under heavy quark spin transformations. The four-vector v^μ is the four-velocity of the heavy meson. The low-momentum strong interaction dynamics of the heavy mesons are described by a Lagrange density of the form [4],

$$\mathcal{L} = -i\mathrm{Tr}\left[\ \overline{H}_v v^\mu \left(\partial_\mu H_v + i H_v V_\mu\right)\ \right] \ -\ g_\pi \mathrm{Tr}\left[\ \overline{H}_v H_v \gamma^\nu \gamma_5 A_\nu\ \right] \quad , \qquad (8)$$

where

$$\overline{H}_v = \gamma^0 H_v^\dagger \gamma^0 \ = \ \left[\ \gamma^\alpha B_\alpha^{*\dagger} \ + \ i\gamma_5 B^\dagger \ \right] \frac{1}{2}(1+\not{v}) \quad , \tag{9}$$

and where the light-meson fields are

$$A_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right) \quad , \quad V_\mu \ = \ \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \quad . \tag{10}$$

The Tr [ ]'s in eq. (8) correspond to traces over Dirac indices, and implicit in eq. (8) are str [ ]'s over the flavor indices. Couplings to Φ_0 [4], such as str [A_μ], have not been included as the Φ_0 will be integrated out of the theory, and such contributions will be included via higher dimension local operators. The axial coupling constant g_π has been constrained to be $g_\pi \sim 0.56$ [12] (or $g_\pi = 0.24$) by the radiative decays $D^* \to D\gamma$ [1], and recent quenched lattice simulations yield $g_\pi \sim 0.42$ [14,15]. The low-momentum strong interaction dynamics of $D^{(*)}$ mesons are described in an analogous way, with heavy quark flavor symmetry dictating that the axial coupling, g_π, for the B's and D's is the same. Further, the dynamics of the anti-heavy mesons can be described in analogous way [4].

III. DECAY CONSTANTS

The heavy meson decay constants, such as f_{B_u}, have been analyzed at the one-loop level in PQχPT by Sharpe and Zhang [4]. Their analysis provides a very clear demonstration of the implementation of PQQCD and PQχPT, and it is worth reviewing their results.

The decay constants of the heavy mesons are defined via the matrix element

$$\langle 0|\overline{q}\gamma^\mu(1-\gamma_5)b|B_q(v)\rangle = -i \ f_{B_q} \ m_B \ v^\mu \quad . \tag{11}$$

In the heavy meson effective field theory, this matrix element receives contributions from both tree-level operators and from diagrams involving light-meson loops. Up to $\mathcal{O}(m_q)$, the local operators that contribute are

$$\begin{aligned} L_\mu^a = & -\frac{\kappa}{2}\mathrm{Tr}\left[\ \gamma_\mu(1-\gamma_5)H_v\xi^\dagger n^a\right] \ - \ \frac{C_1(\Lambda)}{2}\mathrm{Tr}\left[\ \gamma_\mu(1-\gamma_5)H_v\mathcal{M}_+\xi^\dagger n^a\right] \\ & - \ \frac{C_2(\Lambda)}{2}\mathrm{Tr}\left[\ \gamma_\mu(1-\gamma_5)H_v\xi^\dagger\mathcal{M}_+ n^a\right] \ - \ \frac{C_3(\Lambda)}{2}\mathrm{Tr}\left[\ \gamma_\mu(1-\gamma_5)H_v\xi^\dagger n^a\right]\mathrm{str}\left[\ \mathcal{M}_+ \ \right] \quad , \end{aligned} \tag{12}$$

where n^a is a column vector that picks out the light quark associated with the desired current, Λ is the renormalization scale, and $\mathcal{M}_+$ is related to the mass matrix via $\mathcal{M}_\pm \ = \ \frac{1}{2}\left(\xi m_Q\xi \pm \xi^\dagger m_Q\xi^\dagger\right)$. At leading order one finds that $f_{B_q} = \kappa/\sqrt{m_{B_q}}$. At $\mathcal{O}(m_q)$ in PQχPT the decay constants are [4]

$$\begin{aligned} \sqrt{m_{B_q}} \ f_{B_q}^{PQ} = \kappa & \left[1 \ - \ \frac{1+3g_\pi^2}{32\pi^2 f^2}\left(\ 3m_{jq}^2\log\frac{m_{jq}^2}{\Lambda^2} - \frac{1}{3}(2m_{qq}^2 - m_{jj}^2)\log\frac{m_{qq}^2}{\Lambda^2}\right)\right] \\ & + \left(\ C_1^{(R)}(\Lambda) + C_2^{(R)}(\Lambda) \ \right) \ m_q \ + \ 3C_3^{(R)}(\Lambda) \ m_j \quad , \end{aligned} \tag{13}$$

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[1] A more detailed study [13] gives $g_\pi \sim 0.76 \pm 0.2$ or $g_\pi \sim 0.27 \pm 0.07$.

where m_{qq}, m_{jj} and m_{jq} are the masses of light mesons with quark composition $\overline{q}q$, $\overline{j}j$ and $\overline{j}q$ respectively. The $C_i^{(R)}(\Lambda)$ are renormalized constants where both divergences and terms that are analytic in m_q have been absorbed. The expression in eq. (13) holds for all $q = u, d, s, j, l, r$, and agrees with the expression in Ref. [4] when there are three sea quarks, $N_f = 3$. Lattice simulations of $f_{B_{u,d,s}}$ and f_{B_j} at various values of the sea-quark masses, m_j, and various values of the valence masses will allow for the determination of the quantities κ, $C_3^{(R)}(\Lambda)$ and the combination $C_1^{(R)}(\Lambda) + C_2^{(R)}(\Lambda)$. As these counterterms have the same value in QCD and PQQCD, their values can be inserted into the QCD expressions for the decay constants [4,17]

$$\begin{aligned}
\sqrt{m_{B_u}}\ f_{B_u} = \kappa \Bigg[1\ -\ & \frac{1+3g_\pi^2}{32\pi^2 f^2} \left(\frac{3}{2} m_\pi^2 \log\frac{m_\pi^2}{\Lambda^2} + m_K^2 \log\frac{m_K^2}{\Lambda^2} + \frac{1}{6} m_\eta^2 \log\frac{m_\eta^2}{\Lambda^2} \right) \Bigg] \\
& + \left(C_1^{(R)}(\Lambda) + C_2^{(R)}(\Lambda) \right) \overline{m}\ +\ C_3^{(R)}(\Lambda)\ (2\overline{m} + m_s) \\
\sqrt{m_{B_s}}\ f_{B_s} = \kappa \Bigg[1\ -\ & \frac{1+3g_\pi^2}{32\pi^2 f^2} \left(2 m_K^2 \log\frac{m_K^2}{\Lambda^2} + \frac{2}{3} m_\eta^2 \log\frac{m_\eta^2}{\Lambda^2} \right) \Bigg] \\
& + \left(C_1^{(R)}(\Lambda) + C_2^{(R)}(\Lambda) \right) m_s\ +\ C_3^{(R)}(\Lambda)\ (2\overline{m} + m_s) \quad . \qquad (14)
\end{aligned}$$

IV. CHIRAL $1/M_C^2$ CORRECTIONS TO $B \to D^{(*)}$ AT ZERO RECOIL

The matrix elements for the semileptonic decays $B \to De\nu$ and $B \to D^* e\nu$ at zero-recoil are normalized to unity in the heavy quark limit [1]. Further, there are no corrections at zero-recoil of the form $1/m_{c,b}$ by Luke's theorem [18], and the leading corrections in the heavy quark expansion enter at $1/m_c^2$. In addition to contributions from perturbative insertions of local operators in the heavy quark Lagrange density, there are also contributions that receive infrared enhancements from long-distance strong-interactions processes. The largest contribution of this type is from the one-loop diagrams shown in fig. 1 in which the $D^* - D$ mass-splitting is explicitly retained in the $D^{(*)}$ propagator [19]. Recently, quenched lattice QCD simulations of the $B \to D^{(*)}$ form factors have been performed [20,21] but the light-quark mass dependence of the results has yet to be ascertained..

The hadronic matrix elements for $B \to D^{(*)} e\nu$ are

$$\begin{aligned}
\langle D(v') | \overline{c}\gamma^\mu b | B(v) \rangle &= \sqrt{m_B m_D}\ \left[\ h_+(w)(v+v')^\mu\ +\ h_-(w)(v-v')^\mu\ \right] \\
\langle D^*(v',\epsilon) | \overline{c}\gamma^\mu\gamma_5 b | B(v) \rangle &= \sqrt{m_B m_{D^*}}\ \left[\ -i h_{A_1}(w)(w+1)\epsilon^{*\mu} + i h_{A_2}(w)(v\cdot\epsilon^*)v^\mu \right. \\
& \qquad \left. + i h_{A_3}(w)(v\cdot\epsilon^*)v'^\mu\ \right] \\
\langle D^*(v',\epsilon) | \overline{c}\gamma^\mu b | B(v) \rangle &= \sqrt{m_B m_{D^*}}\ h_V(w)\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta \quad , \qquad (15)
\end{aligned}$$

where v_μ is the four-velocity of the initial-state B-meson, v'_μ is the four-velocity of the final-state $D^{(*)}$-meson, $w = v \cdot v'$ and $h_\pm$, h_V and $h_{A_{1,2,3}}$ are the six independent form factors that contribute. In the heavy quark limit $h_-(w) = h_{A_2}(w) = 0$ and $h_+(w) = h_V(w) = h_{A_{1,3}}(w) = \xi(w)$ where $\xi(w)$ is the universal function for the $B^{(*)} \to D^{(*)}$ decays known as the Isgur-Wise function. The matrix element is reproduced in the heavy meson chiral lagrangian by an operator of the form

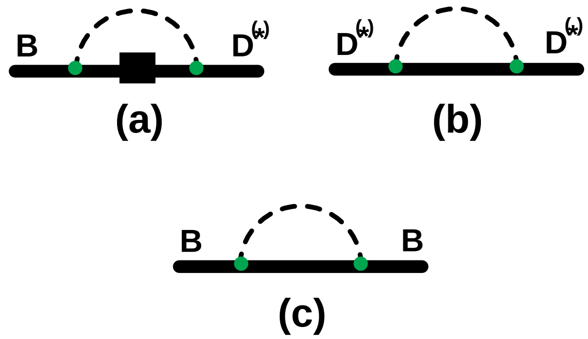

FIG. 1. *One-loop diagrams that contribute to the matrix elements for $B \to D^{(*)}$. The solid lines denote mesons containing a heavy quark, while the dashed lines denote light mesons. The solid square corresponds to an insertion of the weak interaction vertex, while the small solid circles denote a strong interaction vertex $\propto g_\pi$. Diagram (a) is a vertex correction, while diagrams (b) and (c) correspond to wavefunction renormalization.*

$$\overline{c}\gamma^\mu(1-\gamma_5)b \to -\xi(w)\ \mathrm{Tr}\left[\ \overline{H}^{(c)}_{v'}\gamma^\mu(1-\gamma_5)H^{(b)}_v\ \right] \quad , \tag{16}$$

where $\xi(w)$ is the Isgur-Wise function, and we have retained the flavor labels on the H fields.

The lowest order heavy-quark-spin-symmetry breaking operator in the heavy quark chiral Lagrangian is dimension three due to the chromomagnetic moment interactions of the charm quark with the light degrees of freedom, $\sim \overline{c}\ \sigma^{\mu\nu}G^{\mu\nu}c/m_c$ (we neglect the contribution from the b-quark chromomagnetic interactions, i.e. $m_b \gg m_c$). This interaction gives rise to a mass-splitting between the D^* and D at order $1/m_c$ in the heavy meson chiral Lagrangian, which after performing the Dirac trace, is

$$\delta\mathcal{L}^{(c)} = \Delta^{(c)} D^{*\dagger}_\alpha D^{*\alpha} \quad . \tag{17}$$

As this is the lowest dimension operator that breaks the heavy quark symmetries, it provides the dominant long-distance contribution to the $1/m_c^2$ corrections to the matrix elements in eq. (15) [19]. Randall and Wise [19] found a contribution of the form $\left(\Delta^{(c)}\right)^2 \log\left(m_\pi^2/\Lambda^2\right) + ...$ at $w=1$ from the pion loop-diagrams shown in fig. 1. In order to exactly compensate the renormalization scale dependence, Λ, the $1/m_c^2$ operators that are independent of m_q will combine together to give a m_q-independent contribution $X_+(\Lambda)$ at $w=1$ for the $B \to D$ decay and $X_A(\Lambda)$ at $w=1$ for the $B \to D^*$ decay. The form factors at zero-recoil are modified at $\mathcal{O}\left(1/m_c^2\right)$ by the one-loop diagrams in fig. 1 to become

$$\begin{aligned}
h_+^{(B_q)PQ}(1) &= 1\ +\ X_+(\Lambda)\ +\ \frac{g_\pi^2}{16\pi^2 f^2}\left[\ 3F_{jq}\ -\ \frac{1}{3}F_{qq}\ -\ \frac{1}{3}\left(m_{qq}^2 - m_{jj}^2\right)R_{qq}\ \right]\\
h_+^{(B_j)PQ}(1) &= 1\ +\ X_+(\Lambda)\ +\ \frac{g_\pi^2}{6\pi^2 f^2}\ F_{jj}\\
h_{A_1}^{(B_q)PQ}(1) &= 1\ +\ X_A(\Lambda)\ +\ \frac{g_\pi^2}{48\pi^2 f^2}\left[\ 3\overline{F}_{jq}\ -\ \frac{1}{3}\overline{F}_{qq}\ -\ \frac{1}{3}\left(m_{qq}^2 - m_{jj}^2\right)\overline{R}_{qq}\ \right]\\
h_{A_1}^{(B_j)PQ}(1) &= 1\ +\ X_A(\Lambda)\ +\ \frac{g_\pi^2}{18\pi^2 f^2}\ \overline{F}_{jj} \quad ,
\end{aligned} \tag{18}$$

where we have used the shorthand $F_{ab} = F(m_{ab}, \Delta^{(c)}, \Lambda)$, $\overline{F}_{ab} = F(m_{ab}, -\Delta^{(c)}, \Lambda)$, $R_{ab} = R(m_{ab}, \Delta^{(c)}, \Lambda)$ and $\overline{R}_{ab} = R(m_{ab}, -\Delta^{(c)}, \Lambda)$. The expressions in eq. (18) are true for $q = u, d, s$. The loop functions F and R are

$$
\begin{aligned}
F(m, \delta, \Lambda) &= \delta^2 \log\left(\frac{m^2}{\Lambda^2}\right) - 2(2m^2 - \delta^2) - \delta\sqrt{\delta^2 - m^2} \log\left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\epsilon}}\right) \\
&\quad + \frac{2m^2}{\delta}\left(\pi m - \sqrt{\delta^2 - m^2} \log\left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\epsilon}}\right)\right) \\
&\rightarrow \delta^2 \log\left(\frac{m^2}{\Lambda^2}\right) + \mathcal{O}\left(\delta^3\right) \\
R(m, \delta, \Lambda) &= \frac{3\pi m}{\delta} - 6 - \frac{3(\delta^2 - 2m^2)}{2\delta\sqrt{\delta^2 - m^2}} \log\left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\epsilon}}\right) \\
&\rightarrow \frac{\delta^2}{m^2} + \mathcal{O}\left(\delta^3\right) \quad . \qquad (19)
\end{aligned}
$$

It is interesting to note that in the $\Delta^{(c)} \ll m_{qq}$ limit, there is a contribution to the $h_i^{(B_q)PQ}(1)$ of the form $(m_{qq}^2 - m_{jj}^2)/m_{qq}^2$. This is analogous to the enhanced chiral logarithms found by Sharpe [5] that contribute to the light meson masses and decay constants. Once lattice simulations have determined $X_+(\Lambda)$ and $X_A(\Lambda)$, they can be inserted into the QCD expressions for $h_+(1)$ and $h_{A_1}(1)$ for $B_{u,d,s}$ to obtain the zero-recoil matrix elements in QCD,

$$
\begin{aligned}
h_+^{(B_u)}(1) &= 1 + X_+(\Lambda) + \frac{g_\pi^2}{16\pi^2 f^2}\left[\frac{3}{2}F_\pi + F_K + \frac{1}{6}F_\eta\right] \\
h_+^{(B_s)}(1) &= 1 + X_+(\Lambda) + \frac{g_\pi^2}{16\pi^2 f^2}\left[2F_K + \frac{2}{3}F_\eta\right] \\
h_{A_1}^{(B_u)}(1) &= 1 + X_A(\Lambda) + \frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{3}{2}\overline{F}_\pi + \overline{F}_K + \frac{1}{6}\overline{F}_\eta\right] \\
h_{A_1}^{(B_s)}(1) &= 1 + X_A(\Lambda) + \frac{g_\pi^2}{48\pi^2 f^2}\left[2\overline{F}_K + \frac{2}{3}\overline{F}_\eta\right] \quad , \qquad (20)
\end{aligned}
$$

where we have used $F_Y = F(m_Y, \Delta^{(c)}, \Lambda)$ and $\overline{F}_Y = F(m_Y, -\Delta^{(c)}, \Lambda)$. Keeping only the contribution from pion-loops and counterterms we recover the results of Ref. [19].

V. CHIRAL CORRECTIONS TO THE $B \rightarrow D$ ISGUR-WISE FUNCTION

While the Isgur-Wise function $\xi(w)$ in eq. (16) is normalized to unity at zero recoil, $\xi(1) = 1$, away from zero recoil its value depends upon the light quark masses. In QCD the leading non-analytic contributions to the Isgur-Wise function for $B_{u,d}$ decays, $\xi_{B_{u,d}}(w)$, and the Isgur-Wise function for B_s decays, $\xi_{B_s}(w)$, of the form $m_q \log m_q$ have been computed [22,23]. In addition, the contributions of this form arising in quenched QCD (QQCD) have also been computed [24]. In the heavy quark limit, matrix elements of the operator $\overline{c}\gamma^\mu(1-\gamma_5)b$ between B and $D^{(*)}$ states are reproduced, up to $\mathcal{O}(m_q)$, by

$$\overline{c}\gamma^\mu(1-\gamma_5)b \to -\xi(w)\ \mathrm{Tr}\left[\ \overline{H}^{(c)}_{v'}\gamma^\mu(1-\gamma_5)H^{(b)}_v\right]\ -\ \eta_1(w,\Lambda)\mathrm{Tr}\left[\ \overline{H}^{(c)}_{v'}\gamma^\mu(1-\gamma_5)H^{(b)}_v\mathcal{M}_+\ \right]$$
$$-\ \eta_2(w,\Lambda)\ \mathrm{str}\left[\ \mathcal{M}_+\ \right]\mathrm{Tr}\left[\ \overline{H}^{(c)}_{v'}\gamma^\mu(1-\gamma_5)H^{(b)}_v\ \right]\quad , \tag{21}$$

where we use notation similar to Ref. [23] for the higher order counterterms, $\eta_{1,2}$, and it is understood that flavor indices are super-traced over. In addition heavy quark symmetry requires that $\eta_{1,2}(1,\Lambda)=0$ The one-loop diagrams shown in fig. 1 and the contributions from eq. (21) give Isgur-Wise functions in QCD at $\mathcal{O}\left(m_q\right)$

$$\xi_{B_{u,d}}(w) = \xi(w)\ \left[\ 1+\frac{g_\pi^2}{8\pi^2f^2}\left[\ r(w)-1\ \right]\left[\ \frac{3}{2}m_\pi^2\log\frac{m_\pi^2}{\Lambda^2}+m_K^2\log\frac{m_K^2}{\Lambda^2}+\frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\Lambda^2}\ \right]\ \right]$$
$$+\ \eta_1^{(R)}(w,\Lambda)\ \overline{m}\ +\ \eta_2^{(R)}(w,\Lambda)\left(\ 2\overline{m}\ +\ m_s\ \right)$$
$$\xi_{B_s}(w) = \xi(w)\ \left[\ 1+\frac{g_\pi^2}{8\pi^2f^2}\left[\ r(w)-1\ \right]\left[\ 2m_K^2\log\frac{m_K^2}{\Lambda^2}+\frac{2}{3}m_\eta^2\log\frac{m_\eta^2}{\Lambda^2}\ \right]\ \right]$$
$$+\ \eta_1^{(R)}(w,\Lambda)\ m_s\ +\ \eta_2^{(R)}(w,\Lambda)\left(\ 2\overline{m}\ +\ m_s\ \right)\quad , \tag{22}$$

where the w-dependent function is [22,23]

$$r(w) = \frac{1}{\sqrt{w^2-1}}\ \log\left[w\ +\ \sqrt{w^2-1}\right]\quad . \tag{23}$$

$\eta_{1,2}^{(R)}$ are renormalized constants with $\eta_{1,2}^{(R)}(1,\Lambda)=0$.

In PQχPT, the Isgur-Wise functions for mesons composed of a b-quark and an anti-valence quark, $b\overline{u}$, $b\overline{d}$, or $b\overline{s}$, at one-loop order and in the limit of isospin symmetry, are

$$\xi^{PQ}_{B_q}(w) = \xi(w)\ \left[\ 1+\frac{g_\pi^2}{8\pi^2f^2}\left[\ r(w)-1\ \right]\left[3m_{jq}^2\log\frac{m_{jq}^2}{\Lambda^2}-\frac{1}{3}(2m_{qq}^2-m_{jj}^2)\log\frac{m_{qq}^2}{\Lambda^2}\right]\ \right]$$
$$+\ \eta_1^{(R)}(w,\Lambda)\ m_q+3\ \eta_2^{(R)}(w,\Lambda)\ m_j\quad , \tag{24}$$

where $q=u,d,s$. The Isgur-Wise function for mesons comprised of a b-quark and an anti-sea quark, j, is

$$\xi^{PQ}_{B_j}(w) = \xi(w)\left[1+\frac{g_\pi^2}{3\pi^2f^2}\left[\ r(w)-1\ \right]m_{jj}^2\log\frac{m_{jj}^2}{\Lambda^2}\ \right]+\left(\eta_1^{(R)}(w,\Lambda)+3\eta_2^{(R)}(w,\Lambda)\right)m_j\ . \tag{25}$$

As required, the expression in eq. (22) reduces to the expression in eq. (25) when $m_\pi=m_K=m_\eta=m_{jj}$ and $m_{u,d,s}=m_j$. By determining $\xi^{PQ}_{B_q}(w)$ and $\xi^{PQ}_{B_j}(w)$ in lattice simulations and fitting the quantities $\xi(w)$ and $\eta_{1,2}(w,\Lambda)$, one can then use the QCD expressions for $\xi_{B_{u,d,s}}(w)$ in eq. (22) to perform the extrapolation in m_q.

VI. RADIATIVE TRANSITIONS: $D^*\to D\gamma$

The widths of the radiative decays $D^*\to D\gamma$ are relatively well measured. These decays are quite interesting as they have allowed for a determination of the axial coupling constant g_π, in combination with the leading order magnetic transition moment counterterm [12,13].

The value for g_π that has been extracted from these decays is significantly smaller [12,13] than the value expected in the naive quark model.

The width for radiative decays is given in terms of a total radiative matrix element μ_a,

$$\Gamma(D_a^* \to D_a\gamma) = \frac{\alpha}{3}\ |\mu_a|^2\ |\mathbf{k}_\gamma|^3 \quad , \tag{26}$$

where the subscript "a" denotes the flavor of the $D^{(*)}$ meson. At leading order there are contributions that are independent of the light quark masses arising from the coupling of the photon to the light degrees of freedom, and to the heavy quark. At next to leading order there is a contribution of the form $\sqrt{m_q}$ resulting from the one-loop diagrams shown in fig. 2. In the heavy quark limit, the transition moments in QCD are

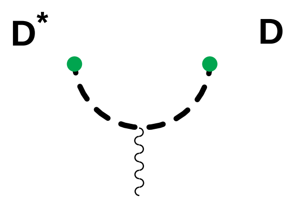

FIG. 2. *One-loop diagrams that contribute to the radiative transitions $D^* \to D\gamma$. The solid lines denote mesons containing a heavy quark, the dashed lines denote light mesons and the small solid-circle denotes a strong interaction $\propto g_\pi$.*

$$\begin{aligned}
\mu_u &= \frac{2}{3m_c} + \frac{2}{3}\beta - \frac{g_\pi^2 m_K}{4\pi f^2} - \frac{g_\pi^2 m_\pi}{4\pi f^2} \\
\mu_d &= \frac{2}{3m_c} - \frac{1}{3}\beta + \frac{g_\pi^2 m_\pi}{4\pi f^2} \\
\mu_s &= \frac{2}{3m_c} - \frac{1}{3}\beta + \frac{g_\pi^2 m_K}{4\pi f^2} \quad ,
\end{aligned} \tag{27}$$

where β is the m_q-independent counterterm contribution from the light degrees of freedom.

In PQQCD the calculations are analogous to those of QCD, and one finds that

$$\begin{aligned}
\mu_u^{PQ} &= \frac{2}{3m_c} + \frac{2}{3}\beta - \frac{g_\pi^2 m_{ju}}{2\pi f^2} \\
\mu_d^{PQ} &= \frac{2}{3m_c} - \frac{1}{3}\beta + \frac{g_\pi^2 m_{ju}}{4\pi f^2} \\
\mu_s^{PQ} &= \frac{2}{3m_c} - \frac{1}{3}\beta + \frac{g_\pi^2 m_{js}}{4\pi f^2} \\
\mu_j^{PQ} &= \frac{2}{3m_c} + \frac{2}{3}\beta - \frac{g_\pi^2 m_{jj}}{2\pi f^2} \quad ,
\end{aligned} \tag{28}$$

where the coefficient β is the same in QCD and PQQCD. A fit to lattice calculations of these matrix elements will provide a determination of β, and consequently an estimate of the radiative matrix elements for the physical values of m_q.

VII. MATRIX ELEMENTS OF ISOVECTOR TWIST-2 OPERATORS

Recently, it has been realized that the chiral corrections to the matrix elements of isovector twist-2 operators in the nucleon and octet-baryons can be computed in chiral perturbation theory [25–29]. Such corrections systematically incorporate the long-distance strong interaction contributions to the moments of the parton distributions. While it is not possible to perform deep-inelastic-scattering (DIS) from hadrons containing heavy quarks, it is possible to consider the parton distributions of heavy hadrons from a theoretical standpoint, and further it is likely that lattice studies of these distributions will be significantly easier than analogous studies in nucleons.

In QCD the twist-2 isovector operators have the form

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n} = \frac{1}{n!}\ \overline{q}\ \lambda^a\ \gamma_{\{\mu_1}\left(i\stackrel{\leftrightarrow}{D}_{\mu_2}\right)\ \dots\left(i\stackrel{\leftrightarrow}{D}_{\mu_n\}}\right)\ q\ -\ \text{traces}\quad , \tag{29}$$

where the $\{...\}$ denotes symmetrization on all Lorentz indices, and λ^a are $SU(3)$ Gell-Mann matrices. In PQQCD the operator has an analogous form,

$$\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \dots\mu_n} = \frac{1}{n!}\ \overline{Q}\ \overline{\lambda}^a\ \gamma_{\{\mu_1}\left(i\stackrel{\leftrightarrow}{D}_{\mu_2}\right)\ \dots\left(i\stackrel{\leftrightarrow}{D}_{\mu_n\}}\right)\ Q\ -\ \text{traces}\quad , \tag{30}$$

where $\overline{\lambda}^a$ is a super Gell-Mann matrix. As an example, the $\overline{\lambda}^3$ matrix has entries $\mathrm{diag}(1,-1,0,1,-1,0,1,-1,0)$. Forward matrix elements of $\mathcal{O}^{(n),3}_{\mu_1\mu_2\ \dots\mu_n}$ between heavy meson states are reproduced up to $\mathcal{O}(m_q)$ by forward matrix elements of

$$\begin{aligned}\mathcal{O}^{(n),3}_{\mu_1\mu_2\ \dots\mu_n}\ \rightarrow\ -\frac{1}{2}v_{\mu_1}v_{\mu_2}...v_{\mu_n}\Big[\ & t^{(n)}\mathrm{Tr}\left[\ \overline{H}_v H_v \overline{\lambda}^3_{\xi+}\ \right]\ +\ s_1^{(n)}(\Lambda)\mathrm{Tr}\left[\ \overline{H}_v H_v \mathcal{M}_+ \overline{\lambda}^3_{\xi+}\ \right]\\ &+\ s_2^{(n)}(\Lambda)\mathrm{Tr}\left[\ \overline{H}_v H_v \overline{\lambda}^3_{\xi+}\mathcal{M}_+\ \right]\ +\ s_3^{(n)}(\Lambda)\mathrm{Tr}\left[\ \overline{H}_v H_v \overline{\lambda}^3_{\xi+}\ \right]\mathrm{str}\left[\ \mathcal{M}_+\ \right]\Big]\quad ,\end{aligned} \tag{31}$$

where $\overline{\lambda}^3_{\xi+}\ = \frac{1}{2}\left(\xi^\dagger\overline{\lambda}^3\xi + \xi\overline{\lambda}^3\xi^\dagger\right)$, and it is understood that flavor indices are super-traced over. At $\mathcal{O}\left(m_q\right)$ in the chiral expansion the forward matrix elements receive contributions from the operators in eq. (31) and the one-loop diagrams shown in fig. 3,

$$\begin{aligned}\langle B_u|\mathcal{O}^{(n),3}_{\mu_1\mu_2\ \dots\mu_n}|B_u\rangle^{PQ} =\ & v_{\mu_1}v_{\mu_2}...v_{\mu_n}\Big[\ t^{(n)}\left(\ 1-\frac{3(1+3g_\pi^2)}{16\pi^2 f^2}m_{ju}^2\log\frac{m_{ju}^2}{\Lambda^2}\ \right)\\ &\qquad +\ \left(\ s_1^{(R)(n)}(\Lambda)\ +\ s_2^{(R)(n)}(\Lambda)\right)\ \overline{m}\ +\ 3s_3^{(R)(n)}(\Lambda)\ m_j\ \Big]\\ \langle B_j|\mathcal{O}^{(n),3}_{\mu_1\mu_2\ \dots\mu_n}|B_j\rangle^{PQ} =\ & v_{\mu_1}v_{\mu_2}...v_{\mu_n}\Big[\ t^{(n)}\left(\ 1-\frac{3(1+3g_\pi^2)}{16\pi^2 f^2}m_{jj}^2\log\frac{m_{jj}^2}{\Lambda^2}\ \right)\\ &\qquad +\ \left(\ s_1^{(R)(n)}(\Lambda)\ +\ s_2^{(R)(n)}(\Lambda)\ +\ 3s_3^{(R)(n)}(\Lambda)\ \right)\ m_j\ \Big]\quad ,\end{aligned} \tag{32}$$

for $n=$ odd and $n>1$. In the loop calculations we have only retained the non-analytic contributions, and the $s_i^{(R)(n)}$ are renormalized counterterms. For $n=1$ the matrix element is absolutely normalized since $\mathcal{O}^{(1),3}_{\mu_1}$ is the isovector charge operator. Matrix elements in the other meson states can be found straightforwardly from those in eq. (32). From PQQCD lattice simulations one envisages that the constants $t^{(N)}$, $s_3^{(R)(n)}(\Lambda)$ and the combination $s_1^{(R)(n)}(\Lambda)\ +\ s_2^{(R)(n)}(\Lambda)$ can be individually determined and then inserted into the analogous

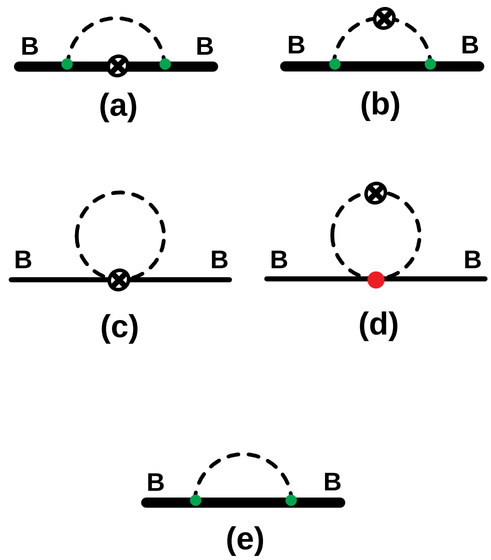

FIG. 3. *One-loop diagrams that contribute to the matrix elements of the isovector twist-2 operators in mesons containing a heavy quark. The solid lines denote mesons containing a heavy quark, while the dashed lines denote light mesons. The crossed-circle denotes an insertion of* $\mathcal{O}^{(n),a}_{\mu_1\mu_2\ \ldots\mu_n}$ *while the small solid-circle denotes a strong interaction* $\propto g_\pi$*.*

expressions in QCD to determine the moments of combinations of parton distributions in heavy mesons. In QCD I find the matrix element between B_u states to be

$$\langle B_u|\mathcal{O}^{(n),3}_{\mu_1\mu_2\ \ldots\mu_n}|B_u\rangle = v_{\mu_1}v_{\mu_2}...v_{\mu_n}\left[t^{(n)}\left(1-\frac{(1+3g_\pi^2)}{16\pi^2 f^2}\left(2m_\pi^2\log\frac{m_\pi^2}{\Lambda^2} + m_K^2\log\frac{m_K^2}{\Lambda^2}\right)\right)\right.$$
$$\left. + \left(s_1^{(R)(n)}(\Lambda) + s_2^{(R)(n)}(\Lambda) + 2s_3^{(R)(n)}(\Lambda)\right)\ \overline{m} + s_3^{(R)(n)}(\Lambda)\ m_s \right] . \tag{33}$$

VIII. CONCLUSIONS

In this work I have computed several observables in mesons containing heavy quarks at one-loop level in partially quenched heavy quark chiral perturbation theory and in ordinary heavy quark chiral perturbation theory. In order to determine these observables from numerical lattice QCD simulations, an extrapolation in the quark masses from those that can be simulated on the lattice down to their physical values is required. The recent progress in understanding partially quenched QCD has provided an efficient pathway to perform such extrapolations. The counterterms that contribute to partially quenched observables are the same as those that contribute to the analogous QCD observables, and hence partially quenched simulations along with the light-quark mass dependence about the chiral limit are sufficient to recover the QCD prediction to some order in the chiral expansion.

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